§34. Analytical Formula of the Thermal Conductivity due to Given Electrostatic and Electromagnetic Fluctuations-1

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In given electrostatic and electromagnetic fluctuations, there may be cases that the transport of guiding center test particles by the fluctuations is regarded as a diffusion process due to stochastic instability of orbits. In such cases, an analytical formula of the thermal conductivity is derived, which may give the most simple evaluation of the transport process in given electrostatic and electromagnetic fluctuations. The theory is based on

1. treating the deterministic equations of motion as Stochastic Differential Equations (SDE),
2. obtaining the formal solution of the SDE in the cylindrical geometry (r, θ, ζ),
3. assuming the statistical properties of parts related to the fluctuating electro-magnetic field as Gaussian,
4. renormalizing the Lagrangian autocorrelation function by using the perturbed particle orbits (note that in the qusi-linear approximation, unperturbed particle orbits are used),
5. assuming the Lagrangian autocorrelation function to have a finite decorrelation time,
6. relating the Lagrangian autocorrelation function in the θ direction to that in the r direction,
7. relating the mono-energetic diffusion coefficient to the Lagrangian autocorrelation function through the standard method,
8. realizing the stochastic instability by treating discrete parallel wave numbers as the continuous quantity,
9. approximately solving the resultant nonlinear equation of the mono-energetic running diffusion coefficient,
10. integrating the mono-energetic running diffusion coefficient in the velocity space.

The resultant mono-energetic running diffusion coefficient is expressed as

$$
\begin{aligned}
D_r \sim\ & \frac{L_{\|}}{4\pi} \sum_m \left\langle v_{\|} \left[\frac{\delta B_{rmk_{\|}}}{B} \right]^2 \right\rangle_{k_{\|}} \\
& \times \left\{ \mathrm{Tan}^{-1} \left[\frac{\delta k_{\|} v_{\|} - \hat{\omega}_m^{(\delta B)}}{\overline{k}_r^2 D_r} \right] \right. \\
& \left. + \mathrm{Tan}^{-1} \left[\frac{\delta k_{\|} v_{\|} + \hat{\omega}_m^{(\delta B)}}{\overline{k}_r^2 D_r} \right] \right\} \\
& + \frac{L_{\|}}{4\pi} \sum_m \left\langle \frac{1}{v_{\|}} \left[\frac{\delta E_{\theta mk_{\|}}}{B} \right]^2 \right\rangle_{k_{\|}} \\
& \times \left\{ \mathrm{Tan}^{-1} \left[\frac{\delta k_{\|} v_{\|} - \hat{\omega}_m^{(\delta E)}}{\overline{k}_r^2 D_r} \right] \right. \\
& \left. + \mathrm{Tan}^{-1} \left[\frac{\delta k_{\|} v_{\|} + \hat{\omega}_m^{(\delta E)}}{\overline{k}_r^2 D_r} \right] \right\}
\end{aligned}
$$

where m is the poloidal mode number, $\langle Q \rangle_{k_{\|}}$ is the averaged value of Q with repsect to the prallel wave number $k_{\|}$, $L_{\|}$ is the corrrelation length in the direction parallel to the unperturbed magnetic field, $\delta k_{\|}$ is the width of the parallel wave numbers contributing to the diffusion, $\overline{k}_r$ is the typical radial wave number,

$$
\begin{aligned}
\hat{\omega}_m^{(\delta B)} &\equiv \left\langle \omega_{mk_{\|}}^{(\delta B)} \right\rangle_{k_{\|}} - m\omega_{E\times B}, \\
\hat{\omega}_m^{(\delta E)} &\equiv \left\langle \omega_{mk_{\|}}^{(\delta E)} \right\rangle_{k_{\|}} - m\omega_{E\times B},
\end{aligned}
$$

and $\omega_{E\times B}$ is the $\vec{E} \times \vec{B}$ drift frequency, and $\omega_{mk_{\|}}^{(\delta B)}$ and $(\omega_{mk_{\|}}^{(\delta E)})$ are the frequency of the fluctuations with m and $k_{\|}$ for electrostatic and electromagnetic fluctuations, respectively. Note that

$$
\begin{aligned}
D_r(-v_{\|}, \hat{\omega}_m^{(\delta B)}, \hat{\omega}_m^{(\delta E)}) &= D_r(v_{\|}, \hat{\omega}_m^{(\delta B)}, \hat{\omega}_m^{(\delta E)}), \\
D_r(v_{\|}, -\hat{\omega}_m^{(\delta B)}, -\hat{\omega}_m^{(\delta E)}) &= D_r(v_{\|}, \hat{\omega}_m^{(\delta B)}, \hat{\omega}_m^{(\delta E)})
\end{aligned}
$$